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METHODS

IN

ELEMENTARY SCHOOL STUDIES

The LMC Co.

METHODS

IN

ELEMENTARY SCHOOL STUDIES

A BRIEF OUTLINE

BY

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"Theory alone is inspiring, but vague; practice alone is definite, but formal; theory combined with practice is inspiring, rational, and definite."

PREFACE.

A KNOWLEDGE of specific methods is a necessary preparation to the art of teaching, because —

1. Method is the medium between teacher and pupil. It is not the question whether the teacher understands the rationale of the processes that concerns the child, but the validity or the falsity of the method employed. Teaching, or the process whereby one mind causes knowledge to arise in another, is an art; the teacher, during the performance of his task, is an artisan; and as such he should know how to use his tools, namely, his methods.

2. The mere study of the science of education does not of itself insure the mastery of methodology any more than theoretic knowledge in any other field carries with it also skill in application. Even Ruskin, with all his keen insight, would scarcely be regarded as a competent teacher of painting. And why not? "There is in the master's profession," observes J. G. Fitch, "the same difference which is observable in all other human employments between the skilled and the unskilled practitioner, and . . . this difference depends in a large measure on a knowledge of the best rules

and methods which have to be used. It is easy to say of the schoolmaster, 'nascitur non fit,' and to give this as a reason why all training and study of method are superfluous. But we do not reason thus in regard to any other profession, even to those in which original power tells most, and in which the mechanic is most easily distinguishable from the inspired artist."

3. The study of the science of education and the subsequent deduction of methods by the individual teacher will not satisfy the demands of a common-sense pedagogy, because —

(*a*) The average man fails to trace back deductions to their underlying principles for verifications, and rests content in the belief that his practice is in accord with his theory. The teacher is no exception, — he masters his science, then, blissfully unconscious, proceeds with wrong applications.

(*b*) It takes years of unaided effort to build up a satisfactory method, and if we take into consideration the fact that the number of years thus spent in preparation is greater than the average teacher devotes to his profession, it will be readily seen that the benefits accruing to the teacher in power and breadth, not to mention freedom, will scarcely offset the injustice to which the child is subjected during that period, — and the child has a right to object to being made the subject of experiment.

4. Nor is the claim valid that training in specific methods unfits the teacher for spontaneous, independ-

ent, characteristic work. "No natural educator," says W. Rein, "is so gifted through divine favor from the beginning as to be able to reach the highest results entirely without the aid of methodical schooling, and there will never be a method so wonderful as to be able to supplant the power of a strong personality. . . . This truth must nevertheless be advanced against the scorners of all method: even the most happily constituted nature, the teacher by divine grace, is not restricted nor rendered ineffective by the directions of method; on the contrary, his activity is promoted and insured of its effectiveness." There is indeed no more valid reason for the claim that the study of method tends to suppress the individuality of the teacher than there was in the claim that the teaching of penmanship by printed models would crush the individuality of the child's handwriting. There is more in human nature, in human character, in the human mind than what is put there by the pedagogue. Nay, more, nature will have its say in spite of him; so that the student with a taste for investigation will not renounce his work simply because such work has been facilitated for him; while he who is not so inclined will certainly not be assisted by the fact that the study of methods formed no part of his preparation.

ever, is not an end in itself, but a preparation for the more serious and important work of inducing the phonic elements of our language and the powers of the letters. In the subsequent lessons, therefore, the child's attention is directed to the comparison of words having common characteristics. These, in obedience to the mind's tendency, gradually dissociate themselves from the words of which they form parts, and become generalized into definite symbol-sounds.

As a brief illustration of the method: The child who has learned to recognize as symbols some such words as

tar	fat
car	fan
far	for

will experience but little difficulty in abstracting the common elements and in assigning to each its appropriate sound.

Third Stage: Application.

THE ART OF READING.

Aim: Thought.

Means: Symbols.

The ultimate aim of reading is thought-getting; but as this presupposes a recognition of the symbols used in conveying thought, the teacher must bear in mind —

Fundamentals.

1. That the recognition of the thought and the recognition of the symbols are two distinct mental processes, having attention as their common factor.

2. That only a definite amount of attention is available for any given period of time; so that the greater the demand is on the child's attention for symbol-recognition, the less of his attention remains to be expended on thought-getting.

3. That while the aim of reading is constantly held before the child, the teacher should strive to give him a gradually increased proficiency in the recognition of the symbols.

4. That the ideal condition obtains when the means are employed automatically in the attainment of the aim.

Method.

1. Reading by the child.

By throwing the child upon his own resources, he will eventually gain power and self-reliance.

Aids:

(*a*) Syllabication.

Teach the child to focus his attention upon each separate syllable of a word. The syllables united are impregnable; divided, they yield. It is only in this way that words of many syllables are recognized.

(*b*) Comparison.

Most of the syllabic elements the child masters during the first few months of his school life. If he masters also the powers of the letters, he has at his command all the means whereby he may decipher any apparently strange syllable.

2. Reading by the teacher.

The child listens to a model reading lesson, while the teacher takes this opportunity to clear up any vagueness in the content.

3. Thought-getting and repetition.

(*a*) Questions by teacher.

(*b*) Answers by pupils.

The teacher asks a question, making it brief and pointed; the child reads the answer from the book. An opportunity for variety is here presented.

The exercises up to this point have been in the nature of a preparation; now follow:

4. Study by pupil, preferably in the class-room.

5. Reading by pupil.

Model Lessons.

(*a*. Report of a lesson in a 3A grade.)

"The wind played many pranks. He lifted many strong men off their feet, and set them down in the market place. At length, he tired of this kind of fun."

1. During the reading by the child, the following words were developed on the blackboard:

	Known	*Steps*	*Unknown*
(*a*)	bank	ank	
		rank	
		ranks	
			pranks
(*b*)	car	ar	mar-
	let	et	ket
			market

2. The teacher read the selection, questioned upon the meaning of "pranks," and commented upon "He" as referring to wind.

3. *Q.* Why did the wind stop having more fun?

A. "At length ---------- fun."

Q. What does the first story (or sentence) tell you?

A. "The wind ---------- pranks."

Q. Give an example of his pranks.

A. "He lifted ---------- market place."

4. Five minutes were given to the children for silent study of the whole lesson.

5. Reading by the children.

(*b.* Report of a lesson in a 5A grade.)

"A peasant once had a faithful horse, who had grown old and could not serve his master any longer; he did not care, therefore, to provide him with food. So he said to the old horse: 'I really do not want you any more, for you are of no use to me; but if you can prove your strength by bringing me a lion, I will keep you as long as you live. In the meantime, however, just walk out of my stable, and go and make yourself a home in the fields.'"

1. The paragraph was read aloud by a pupil and the pronunciation of the following words was developed:

(*a*) peasant, by comparison with pleasant.

(*b*) provide, syllabicated and accent noted.

The study[1] of unfamiliar words was taken up at this point — a slight variation of the general method.

(*a*) peasant.

Q. What is meant by a "peasant"?

A. (*read from vocabulary*). A peasant is a countryman.

Q. What do we generally call a countryman who works in the fields?

A. A farmer.

Q. Who will give one word for "peasant"?

A. A farmer.

(*b*) provide.

Q. Parents provide children with food. What does "provide" mean?

A. "Provide" means give, supply.

2. Paragraph read by the teacher.

3. Questions asked to develop the thoughts of the paragraph:

Q. Who are the characters in the story?

A. A peasant and a horse.

Q. Why did not the master wish to feed the horse any longer?

A. "He had grown old ---------- any longer."

Q. Had the horse served his master well before he became old?

A. "A peasant once had a *faithful* horse."

[1] An example of correlation.

Q. What did the farmer tell the horse to do?

A. "Just walk out ..... in the fields."

Q. What could the horse do to get back?

A. "If you can prove ..... as you live."

4. Paragraph studied silently by class.

5. Pupils called upon to read, others to reproduce orally.[1]

[1] Correlation.

II. — DICTATION.

Aim: To represent thought in correct written form.

Topics included:

1. Spelling.
2. Homonyms.
3. Capitals.
4. Punctuation.
5. Abbreviations.
6. Paragraphs, Stanzas.
7. Letter Forms.
8. Use of Dictionary.

SPELLING.

Aim: To represent words according to fixed forms.

Spelling, as the art of recording sounds by means of symbols, has for its basis the recognition of elementary sounds and their representative signs or letters. In a phonic language, analysis and comparison suffice for the study of its spelling; in the English language, however, its spelling, though based on rational grounds, presents so many anomalies, that the memory must be called upon to assist the other two processes.

Fundamentals.

1. Analysis is the basic process in spelling — the human mind must first recognize and hold in attention the syllable before it can determine the elements of which it is composed and the symbols which represent them.

2. The analytico-synthetic method should by employed in spelling — word, syllable, letter, word.

3. Each spelling word, when first presented for study, should be introduced in a sentence.

4. Each spelling lesson should include both an oral and a written exercise — the former for study, the latter for testing.

5. Rules for spelling should be learned inductively and applied deductively.

6. The greater the number of the association links as the result of the activity of the various senses, the readier the recall.

Method.

Oral Spelling:

1. The word is first pronounced, then written on the blackboard, syllabicated by vertical lines, and its accents marked.

2. Individual children are then called upon to spell as follows:

(*a*) The word pronounced.

(*b*) The first syllable pronounced.

(*c*) The first syllable spelled.

The remaining syllables having been treated likewise, there follows:

(*d*) The word pronounced.

3. The children with arms extended in front of them, and with fingers pointing to the word, write it several times in the air, while the teacher signals each new letter by counting.

4. The same exercise with eyes closed.

Written Exercise:

1. Words dictated.

2. Words spelled, one pupil at the same time writing them on the blackboard.

3. Errors noted by drawing a line through the word.

4. Papers collected and returned.

5. Corrected forms written above the errors and below the exercise.

6. Study and recitation of corrections.

General Remarks.

1. A dictation should accompany each spelling exercise.

2. Each day's spelling lesson should include the new words of the previous day.

3. The spelling of words whose meanings are not known is a waste of time and energy.

HOMONYMS.

Method.

1. Teach through the medium of the sentence.

2. Teach one word of a group at a time.

3. When the words of a group have been mastered separately, they should then be compared.

4. Test:

   (*a*) Write sentences on blackboard, omitting the homonyms, and require pupils to supply them.

(*b*) Require pupils to give sentences with certain homonyms.

(*c*) Give sentences orally, and require children to spell the homonyms.

CAPITALS.

Steps in Method.[1]

1. Recognition in reading matter.
2. Mastery of rule.
3. Application.

PUNCTUATION.

Punctuation is a device for the ready interpretation of written languages; hence, ability to punctuate demands both a rational and a formal process — the former corresponding to the mental process which determines the separate groups of words, and the latter dealing with the kinds of punctuation marks that should be inserted.

PARAGRAPHING.

Paragraphing, like punctuation, is a device which has a mechanical and a rational side — the former dealing with its printed form, and the latter with the relation which its component sentences bear to itself.

Fundamentals.

1. Necessity for punctuation and for paragraphing is determined by the content of the selections; hence, in

[1] Same for abbreviations and letter forms.

teaching, emphasis should first be laid on the content, then on the form.

2. Since mastery of the thought is a prerequisite to punctuation and to paragraphing, the selections chosen should be adapted to the understanding of the pupils.

3. Punctuation and paragraphing should be learned inductively and applied deductively.

Method.

The Period:

1. John is a good boy.
2. I have a new book.
3. The tree has many leaves.

etc., etc., etc.

The child notices that the first sentence ends in a .

He notices that the second sentence ends in a .

He notices the . at the end of each subsequent sentence.

He concludes[1] that each similar sentence must end in a .

THE PARAGRAPH.

Form:

This knowledge the child obtains by having his attention called to the indention which marks the beginning of each paragraph.

Content:

Successive paragraphs are read and analyzed into their component sentences. Each group is then shown to deal with some particular topic of the theme.

[1] By a similar reasoning he concludes that each sentence must begin with a capital.

Dictation as a Test.

(The Fifth Herbartian Step.)

1. *How the Exercise is Conducted:*

   (*a*) At least one child writes on the blackboard, while the rest write on paper.

   (*b*) The number of words dictated consecutively depends upon the mental capacity of the child and upon the nature of the selection.

   (*c*) Only groups of related words are dictated.

   (*d*) The speed of the dictation is limited by the ability of the majority of the class to take down the exercise verbatim.

   (*e*) No repetition is permissible unless it is demanded by the content of the selection.

2. *Method of Correction:*

   (*a*) Individual children are called upon to read their written exercises, the rest meanwhile inserting omissions.

   (*b*) The blackboard exercise as "proof-sheet," the attention is directed first to the paragraphs as wholes, then to the separate sentences as wholes, and lastly to the elements of each sentence.

   (*c*) Classes that are supplied with dictionaries should use these to look up the correction of misspelled words and the meanings of unknown words.

(*d*) Correction is made by passing a line through the error, rewriting correctly above the word, and in the case of a misspelled word, writing it also below the exercise.

General Directions.

(*a*) Time must be assigned for the study of the corrected forms, and for recitation by several pupils.

(*b*) Each sheet of paper should contain two original dictations; if redictation is deemed advisable, it should be taken on a separate sheet.

(*c*) An oral review of exercises should be had at least once a month, when children are given an opportunity to restudy previous corrections. A recitation should follow.

Model Lesson.[1]

(Report of a lesson in a 6A grade.)

The teacher dictated the following in groups of words included between the vertical lines:

As the fair happened on the following day, | I had intentions of going myself. |

"No, my dear," said my wife, | "our son Moses is a discreet boy, | and can buy and sell to very good advantage." |

[1] The main aim of the lesson was to test the children's knowledge of paragraphs and quotation marks.

As I also had a good opinion of my son's prudence, | I was willing enough to trust the business with him. |

Method of correcting the "Proof-sheet."

Teacher. How many paragraphs in this dictation?

Pupil. Three.

T. Why?

P. Assigns reason.

T. How many complete thoughts in the first paragraph?

P. One.

T. In the second?

P. One.

T. In the third?

P. One.

T. Do you notice any mistakes in the use of the quotation marks? In the use of any other punctuation marks?

P. Assigns reasons for corrections.

T. Do you notice any other mistakes in the first sentence? in the second sentence? in the third sentence?

P. Makes corrections.

Each child next corrected his own exercise, using the blackboard as guide, and studied the correct forms for a subsequent recitation.

The "Proof-sheet."

As the fair happened on the following day, I had intensions of going myself. "No, my dear, said my wife, our son Moses is a discret boy and can buy and sell to very good advantag."

As I also had a good opinion of my son's prudense, I was willing enough to trust the business with him.

1. intentions	5. discreet
2. ¶	6. ,
3. "	7. advantage
4. "	8. prudence

III. — COMPOSITION.

1. *The arrangement of thoughts in their logical order, and*

2. *Their expression in correct language.*

A child has neither the information, the reasoning power, nor the training to write a masterly treatise; but under proper conditions he can be furnished with the information; he can be taught to express it grammatically; and he can be trained to arrange his thoughts logically. It must be borne in mind, however, that the development of a higher faculty through exercise presupposes a particular exercise of a lower faculty.

Fundamentals.

1. The subject-matter and the manner of treatment should be adapted to the child's interests and capacities.

2. Oral composition should be practiced long before an attempt is made at written composition — the penmanship and the spelling are obstacles.

3. Composition should not be subordinated to other studies, but should be made an aim in itself.

4. The child's self-activity should be called into play at every opportunity.

Kinds of Composition:

1. Reproduction of a model (perception).
2. Imitation of a model (imagination).
3. Writing from topical outlines (judgment).
4. Wholly original (reasoning).

Method.

Presentation:

1. 2. 3. (*a*) Models read by teacher and discussed (or subject and treatment discussed for 2 and 3).

(*b*) Orally reproduced by several children.

(*c*) Compositions written, and several read.

4. (*a*) Title written on blackboard.

(*b*) Children volunteer information, and write corrected forms on the blackboard.[1]

(*c*) Attention directed to sequence of sentences and to their division into paragraphs, giving to each of the latter its appropriate heading.

(*d*) Only headings left on blackboard, and children called upon to give oral expression to the thoughts.

(*e*) Compositions written and several read.

[1] This order of presentation is changed as soon as the ability of the children warrants it, by choosing as the initiatory exercise the determination of the topical outline and the volunteering of information by the children.

Correction:

1. Compositions written on blackboard.

2. Criticised by class (see Dictation, Method of Correction, *b* and *d*).

3. Compositions rewritten.

General Remarks:

1. Make haste slowly — quality rather than quantity.

2. The oral work is the most important part of the exercise.

3. Compositions are written on blackboard before or after the regular school session.

4. Home lessons may be assigned in the upper grades.

5. Other compositions besides those written on the blackboard may be criticised, but always in the presence of the child.

Model Lessons.

a. Reproduction:

(Report of a lesson in a 5B grade.)

1. The teacher read the following fable to the class:

THE DOG IN THE MANGER.

There was once a dog who lay all day long in a manger where there was plenty of hay. A horse, a cow, a sheep, and a goat came one by one and wanted to eat the hay. The dog growled at them and would not let them have so much as a mouthful. Then an

ox came and looked in, but the dog growled at him also.

"You selfish fellow!" said the ox, "you cannot eat the hay. Why do you want to keep it all to yourself?"

2. Several pupils were now called upon to reproduce the reading orally and, at the conclusion of each recitation, the class made the necessary corrections.[1]

3. Compositions were written.

4. After a number of pupils had been called upon to read their compositions, which served both as models and as subjects for criticism, the attention of the class was directed to the correction of the "proof-sheet."

5. Compositions that had been criticised either by the class or by the teacher were rewritten.[2]

(Example of a Child's Reproduction.)

THE DOG IN THE MANGER.

There was once a dog who lay all day long in a manger which was full of hay. Soon a horse, a cow, a sheep, and a goat came one by one to eat the hay, but the dog growled at them and would not let them have even a mouthful.

Next, an ox came to eat the hay and the dog growled

[1] In this instance, also, the teacher, by several well-directed questions, led the children to recognize the number of paragraphs into which the reading was divided — another example of correlation.

[2] The teacher himself corrected about one third of the compositions of the class, in the presence of individual pupils.

at him too, but the ox said, "You foolish fellow, you cannot eat hay. Why don't you let somebody else have it then?"

b. Imitation:

(Report of a lesson in a 6B grade.)

1. The teacher read the following to the class:

THE CAT, THE MONKEY, AND THE CHESTNUTS.

A Cat and a Monkey were sitting one day by the hearth, watching some chestnuts which their master had laid down to roast. The chestnuts had begun to burst with the heat, and the Monkey said to the Cat:

"It is plain that your paws were made to pull out those chestnuts. Your paws are, indeed, exactly like our master's hands."

The Cat was greatly flattered by this speech and reached forward for the tempting chestnuts; but scarcely had she touched the hot ashes than she drew back with a cry, for she had burned her paw. She tried again, and made out to get one chestnut; then she pulled another, and a third, though each time she singed the hair on her paws.

When she could pull no more, she turned, and found that the monkey had taken this time to crack the chestnuts and eat them.

2. Several compositions were read, discussed, and criticised.

3, 4, 5, same as in *a*.

(Example of a Child's Imitation.)

THE GIRL, THE BOY, AND THE CANDY.

One day, while a little girl was sitting under a tree eating some candy, a boy came up to her and asked her to get him some water from a spring near by. "Why don't you go yourself?" she asked.

"Because it will taste better if you'll get it for me," he answered. This speech so flattered the little girl that she laid her candy on the grass and ran off to fetch the water.

But, alas! when she returned, there was neither boy nor candy to be seen. That day she learned a lesson she never forgot. It is: "Never allow people to flatter you into doing things which they can do for themselves."

c. Topical outline:

(Report of a lesson in a 7A grade.)

ROBERT FULTON.

1. A talk on the life of Robert Fulton by the teacher, and an oral reproduction by the children.

2. The following topics were written on the blackboard by the teacher:

(*a*) Robert Fulton at home and at school.
(*b*) A youthful invention.
(*c*) His greatest and most successful invention.
(*d*) His death.

3, 4, and 5 same as in *a*.

(Example of a Child's Composition.)

ROBERT FULTON.

Robert Fulton was born in 1765, in Little Britain, Pa. When he was three years old, his father died, and his mother was left to take care of his education. She taught him until he was eight years old. He did not have a great liking for books. He was very much interested in mechanical drawing.

Robert came late to school one day. After being reproved by the schoolmaster, he said, "I was at a shop, hammering a piece of lead into a pencil." Soon after, the children were using the same kind of pencils.

Fulton invented the first steamboat. He called it the *Clermont*, after his friend Livingston's home on the Hudson. Its first trip took place in 1807. It started from New York and sailed up the Hudson River to Albany, which was a distance of one hundred and fifty miles. It took her thirty-two hours to make this trip. People stood along the banks of the Hudson River, saying they didn't think the boat would go. They called it "Fulton's Folly." They were very much surprised when they saw the boat begin to move.

Robert Fulton died in 1815. The world owes a great deal to this man.

d. Original:

(Report of a lesson in an 8A grade.)

1. The pupils, under the supervision of the teacher,

determined upon the following headings, without insisting upon the order of their treatment:

(*a*) What food does.
(*b*) Kinds.
(*c*) Preparation.
(*d*) Necessity.
(*e*) Dangers.

2. The pupils volunteered a number of sentences, each of which, in turn, was first examined for its syntax, and, if approved, was written on the blackboard and examined for written mistakes. It was then assigned to its proper heading.

Sentences Volunteered.

(*a*) Food builds up the body.
Our bodies are constantly wearing away, therefore we must have food to repair them.
Pure food gives pure blood.
(*b*) There are three kinds of food, animal, vegetable, and mineral.
Air is a food.
Some parts of the body need special food, as, for instance, the brain needs phosphorus, and the bones need lime.
(*c*) All food should be well chewed before it is swallowed, or it will cause indigestion.
We cook food to help digestion.

(*d*) If you do not get enough food, your body will become weakened.

We must eat pure food in order to be healthy.

Man cannot live on one kind of food, but must have several kinds mixed in right proportion.

(*e*) There are some things we must avoid, such as tobacco, drugs, and liquor, for they are poisonous.

Liquor is dangerous to the body because it contains alcohol.

We must exercise the muscles in order to get food from the blood.[1]

3. Everything but the headings was erased from the blackboard; and after several pupils had reproduced the subject orally, the class wrote the composition.

4 and 5 same as in *a*.

[1] It need surprise no one to learn that the pupils did not give the sentences in the exact order given above. However, they made a laudable attempt to confine their attention to a single heading at a time — a practice which, if repeated sufficiently often, will tend to form a habit of concentrated thinking in composition writing.

IV. — GRAMMAR.

Aims:

1. *Intellectual discipline.*
2. *Knowledge.*

Grammar deals with certain phases of the content and the form of the sentence. It treats (*a*) of the relations and the functions of the thought-elements, and (*b*) it presents certain forms of expression which usage has imposed upon our language. The standard of the one is in the mind, hence universal and unchangeable; the standard of the other is in custom, and hence subject to change. Analysis, which concerns itself with content, belongs to the former; syntax belongs to the latter. Briefly stated, grammar on its rational side is the psychology of the sentence; on its formal side it is a guide to correct usage.

Fundamentals.

1. The sentence is the basis of the study of grammar.

2. The analysis of a sentence into its elements presupposes:

(*a*) A clear understanding of the thought conveyed.

(*b*) A sufficient degree of mental development.

3. Like all studies that yield to scientific treatment, grammar should be taught inductively and applied deductively.

Method.

Aim: The concept, Sentence.

Place on blackboard a series of unrelated words; as, this, told, birds, farm, a, lived, story, fly, Franklin, boy, on; then direct your pupils' attention to a comparison of these with their rearrangement into the combinations: birds fly, the boy lived on a farm, Franklin told this story, etc.

Generalization (naming) and application should follow in this as in subsequent lessons.

Aim: Analysis of the Simple Sentence.

1.	The sun	shines.
2.	The boy	lived on a farm.
3.	Franklin	told this story.
4.	The city of Boston	is the capital of Massachusetts.
5.	Exercise	must be taken daily.

By means of questions lead your pupils to recognize the two coördinate elements of each sentence, as follows:

Questions by teacher.	Answers by pupils.
1. What shines?	The sun
What does the sun do?	
What is said concerning the sun?	shines.
2. Who lived on a farm?	The boy
What about the boy?	lived on a farm.
3. Who told this story?	Franklin
What did Franklin do, or what does this sentence tell you concerning Franklin?	told this story.
4. What is the capital of Massachusetts?	The city of Boston
What is said of the city of Boston?	is the capital of Massachusetts.
5. What must be taken daily?	Exercise
What concerning exercise?	must be taken daily.

By comparison, the idea is now brought prominently to the pupils' attention that the sentence concerns itself with something of which the rest of the sentence treats.

That "something" is the subject; what is stated concerning the subject is the predicate.[1]

Aim: The concept, Complex Sentence.

1. *A.* I was thus employed.
   B. The enemy discharged several thousand arrows.

We have here two simple sentences which give no evidence of connection or of relationship.

C. While I was thus employed, the enemy discharged several thousand arrows.

By skillful questioning, the pupil can be led to perceive:

1. That the two statements have been incorporated into one (clauses).

2. That the two statements are now not only connected but related — *A* being the occasion of *B* (dependence).

3. That *A* relates to "discharged" (adverbial).

These ideas must now be mastered; and this is done by (*a*) repetition, (*b*) type-sentence.

The type-sentence is of value both as the repository

[1] The difficulty children encounter in discovering the elements of a sentence is due largely to the neglect to emphasize the dependence of these elements upon each other. To obviate this difficulty, the child should be trained to think of the subject-predicate, and of predicate-subject and to recite the subject and predicate together, laying emphasis upon the particular element called for in the answer. Thus in the sentence, "Birds fly," if the subject is asked for, the child recites, "Birds [fly]"; if the predicate, "[Birds] fly."

of the ideas and as the concrete standard with which to compare similar sentences. Hence, every new concept should have a type as its accompaniment.

Repetition is essential to stamp knowledge upon the mind — not the kind of repetition that degenerates into a mechanical process, but the one that calls to its assistance variety, interest, and thought. In conformity with this, the following method is suggested for the study of the type-sentence: Having induced the principle, the pupils, while applying it to a variety of sentences, should be called upon at frequent intervals to apply it to the type-sentence, until they are able to recall readily the sentence and the process.

Model lesson to determine the structural character of a sentence.

To determine the structural character of a sentence, we need to know:

1. The number of clauses composing it.
2. The nature of the relationship existing between these clauses.

Model.

What kind of a sentence is the following?

"Accordingly, when he came to the convention in Independence Hall as a delegate from Virginia, he chose a good seat where he could hear all that was said."

First Step: to determine the number of clauses.

1. (Accordingly) he chose ---------- seat.
2. (when) he came ---------- Virginia.
3. (where) he could hear all.
4. (that) was said.

The idea of the simple sentence is now eliminated.

Second Step: to determine the dependence or independence of the clauses.

1. independent clause.
2. relates to "chose"; dependent clause.
3. relates to "chose"; dependent clause.
4. relates to "all"; dependent clause.

Third Step: Conclusion: Complex Sentence.

The Diagram.

As a graphic representation, it is of great value, especially for beginners, and should be employed for both analysis and synthesis of each sentence.

The Parts of Speech.

Should be taught inductively through the medium of the sentence, with the Formal Steps for guides.

Parsing.

Process: deduction.

Warning: do not invert the order of the deductive process — major premise, minor premise, conclusion.

Examples of parsing:

John bought a new book.

1. Parse "new."

Major premise (adjectives are words which modify nouns or pronouns). Children are credited with this knowledge, and are not called upon to repeat it.

Minor premise: "new" describes "book."

Conclusion: "new" is an adjective.

Briefly: "new" describes "book" and is an adjective.

2. Parse "John."

Proper noun ---------- subject of "bought," and is in the nominative case.

False Syntax.

The recitations and the compositions of the pupils should furnish material for the correction of sentences.

V.—LITERATURE.

A masterpiece of literature may be studied for comprehension, or read for appreciation, or analyzed for criticism.[1] This must be borne in mind in discussing the subject of method in literature.

Fundamentals.

1. As the aim determines the mental processes engaged in its attainment, it also determines the method.

2. As comprehension must precede appreciation, and appreciation must precede criticism, the mental processes engaged in executing these aims bear to each other the relation of the simple to the complex. Hence the method for appreciation includes that for comprehension, and the method for criticism includes that for appreciation.[2]

[1] Criticism is beyond the province of the elementary schools, and will not be discussed here.

[2] The opinion expressed by certain optimistic theorists that a child can appreciate literature that he does not comprehend is about as sound as would be the claim that he can criticise a selection he does not appreciate. As a matter of fact, the child who finds pleasure in listening to the reading of a passage he does not understand does so, not because he appreciates the literature as such, but because he is attracted by its delivery. This view is not poetic, but neither are a great many other truths.

3. Comprehension presupposes a knowledge of the intellectual elements of the composition, and of their relations.

4. Appreciation demands, as conditions to success:

(*a*) Intellectual mastery of the exercise.

(*b*) Feelings sufficiently developed to apperceive the sentiments conveyed in the selection.

(*c*) The imagination awakened to the proper degree of activity.

(*d*) An attractive presentation.

Conclusion: As appreciation is the aim of this study in the elementary schools, a masterpiece will call for either one reading or two. If the selection is one which presents no difficulty to the understanding of the child, one reading, and that for appreciation, is all that is essential; otherwise, two readings will be found necessary — the first for intellectual mastery, and the second for appreciation.

Method.

Preparation. This will include:

(*a*) The name of the author and as much of his life's history as is warranted by the selection. Culture demands this.

(*b*) The occasion of the masterpiece. This gives the proper background and atmosphere to the thoughts and sentiments of the author, and paves the way to a proper apperception. For example, in order that a child may

appreciate the reading of "Old Ironsides," he must first become acquainted with the motive which actuated the author to write this stirring appeal.

Presentation.

(*a*) Comprehension. When a pupil has read through a paragraph carefully and has failed to grasp its meaning, he should search out the cause or causes of his failure and proceed to remove them by a study of:

1. The meanings of unknown words. The individual members of the class, the dictionary, the teacher, and, on rarer occasions, the content, may each become the source of information.

2. Figures of speech. These variations of the ordinary mode of expression serve a most important purpose in literature. They add grace and refinement to thought; they clarify it; they invest it with new charms; they endow it with greater vigor and power. They therefore demand a proper share of the child's attention as living and vital forces of our language.

3. Allusions. These should be regarded in the same light as are unknown words whose meanings are sought. That is, their study should not go beyond what is necessary for a clear comprehension of them. To do more than this tends to introduce breaks in the continuity of the lesson. Brevity is the soul of an allusion.

4. The grammatical construction. The words, phrases, and clauses of each sentence must be grasped in their true relations before the exact meaning of

the author can be known. Hence, grammar is called upon at this stage to complete and to insure thorough comprehension. The extent to which grammar should be utilized in the study of a masterpiece of literature must be measured by the need of it for a clearer understanding of any portion of the selection. Its use must be limited by the consideration that it is of value here only in so far as it aids the child to discover and to strengthen the weak points in his knowledge.[1]

(*b*) Appreciation.[2] While the dictionary, the rhetoric, the reference book, and the grammar, when combined, present a complete array of means for mastering any suitable selection, they cannot, unaided, awaken and develop appreciation. This can be brought about only by the teacher who is gifted with appreciation, power, and insight. Here method concerns itself less with specific directions for the conduct of the lesson, than with the inquiry into the qualifications which a teacher

[1] The question of an immediate rereading of each paragraph that has been studied, and of the final reading of the whole selection for a general intellectual survey, can be answered only by the teacher, who is guided by the varying conditions which confront him.

[2] The appreciative reading of a masterpiece need not follow immediately upon the completion of its study. Indeed, the opinion is ventured that better results will follow if several months are allowed to intervene before entering upon the appreciative reading of a selection that had previously been read for comprehension. In this connection, the plan is suggested of assigning to each grade two classes of masterpieces, — one, easily comprehended, for appreciative reading; the other, of opposite character, for thorough study. The latter may then be read for appreciation during some subsequent term.

must possess, or must strive to acquire, if possible, in order that he may inculcate in the child a desire for good literature.

A teacher who is himself a lover of good literature cannot fail to communicate more or less of this feeling to his pupils; for, "Love beams from the eye, glides from the tongue, and finds its echo in some heart."

A teacher who combines oratorical powers with his love for literature not only awakens and maintains the interest of the pupils in any particular selection, but also instills in them a desire to become acquainted with other selections.

And, finally, if, in addition to his other qualifications, the teacher possesses teaching tact, — if he knows how to adapt his instruction to the child's apperception, interest, and imagination, — he can best attain the aim of this study by merely reading the selection to the class, and by insinuating information, comment, or precept, — in short, whatever his native or acquired tact may suggest.

VI.—NATURE STUDY.

Aims:

1. *To inculcate a feeling of sympathy for nature.*
2. *To learn nature's laws and their application.*

This study is a composite of a number of sciences which fall under the two general divisions of observational and experimental science. To the former belong botany, geology, zoölogy, meteorology, and astronomy; to the latter belong physics and chemistry.

Early man spent a large part of his time in studying the observational sciences. He was interested in the plants, in the soil, in the animals, in the weather, and in the climate, for a very obvious reason — the knowledge was essential to the maintenance of his life. As he advanced in civilization, he devoted a gradually increased share of his attention to physics and chemistry, no longer for the purpose of preserving his life, but to economize effort.

The culture epoch theory, if followed implicitly, would necessitate the study of all the material which interested man in his natural state, without taking into account the changed conditions which confront the student of to-day. This is a fallacy into which the overzealous advocates of nature study have fallen. If

a city child, because of his innate love for the beautiful, is interested in flowers, does it follow that he will be interested in the snake simply because some forefather of his in the remote ages needed that knowledge for his personal safety? If natural phenomena appeal to the child, will the knowledge of cecropia, polyphemus, or cynthia have the same effect? What possible reason, utilitarian, intellectual, æsthetic, or ethical, is there for burdening the child's mind with knowledge of spiders, slugs, toads, and bats?

A little of everything and not much of anything is the motto carried out to an extent little dreamed of even by Comenius. Not only is this study made to include seven sciences, not to mention the sciences which are directly or indirectly, logically or psychologically, sensibly or nonsensically, correlated with it, but each science is loaded down with an overwhelming amount of material, some of which is uninteresting to the child, some is unessential, and some is suitable only for the specialist. If observational science is confined to that portion of earth study and of natural phenomena that is essentially a preparation to the study of geography, and if a little botany is added to it, the child has all he can cover without skimming during the first few years of his school course.

Fundamentals.

1. Observational science should precede the experimental science.

2. Observation should be regarded as a preliminary step to the conception of the causal idea.

3. Basic considerations in the choice of material: the useful, the beautiful, the true.

4. Observation, as well as experiment, should be directed, and preferably, to material with which the child is already familiar.

5. Observation and experiment should be accurate in order to lead to good intellectual habits.

Method.

Observation should place the child in direct contact with the object or phenomenon to be observed. A child can get a clear idea of a plant only by seeing, touching, smelling, and perhaps tasting it. A picture gives him only its form and color, while a verbal description gives him — a lot of words. Again, a prince of India denied the existence of water in solid form, and no amount of persuasion convinced him. He needed to be brought face to face with the phenomenon.

From observation the child passes to experiment, which is the more complex process and includes the former. Here the teacher must be guided by the maxim to have the child make his own inductions and deductions.

Example of induction:

The child is given a ruler, a fulcrum, and several equal weights. He places a weight on each end of the

ruler, which he balances on the fulcrum. He moves one weight nearer the fulcrum and he finds that he must increase the power-distance or the weight in order to preserve the balance. After several observations of like character, he concludes that $P \times Pd = W \times Wd$, a discovery which he proceeds to apply.

VII. — GEOGRAPHY.

Aims:

1. *Reverence.*
2. *Liberal-mindedness.*

A characteristic of modern education is that it aims at the harmonious development of the threefold nature of man — physical, mental, and moral. Another of its characteristics is that, while it conceives it possible to exercise the mental powers through the study of one group of subjects, it insists that certain studies are inherently fitted for particular mental processes. In accordance with this decision, it has assigned nature study to the elementary school primarily for observation, reading for imagination, arithmetic and grammar for reasoning, history for prudential morality, and geography for its highest aim, reverence and liberal-mindedness. Geography, properly taught, yields the material which, if elaborated in accordance with the mind's laws, make possible the attainment of these aims.

Fundamentals.[1]

I. The conditions to reverence are:

1. (*a*) A knowledge of the earth as the dwelling-place of man (utilitarian).

   (*b*) A knowledge of the natural forces that are at work upon it (utilitarian).

[1] Read, in this connection, Fundamentals under History.

2. A knowledge of the relation between the earth and the natural forces (rational).
3. (*a*) Contemplation of the earth beautiful (æsthetic), and
   (*b*) Contemplation of causes and effects, leading up to the conception of the Universal Cause (ethical).

II. The conditions to liberal-mindedness are:

1. (*a*) A knowledge of the physical characteristics of other lands.
   (*b*) A knowledge of their inhabitants.
2. A knowledge of the influence that habitation has upon man.
3. An appreciation of the truth that we are largely the product of our environments.

General Considerations.

1. The map represents the world which the child travels in imagination. Before he can avail himself of its services, however, he must master:

(*a*) The elementary notions of geography.
(*b*) The manner of representing and of interpreting these notions.

2. Elementary geographical notions should result from observation or experience.

3. Induction is the proper method for the teaching of geography.

4. Ability to interpret the map rationally is of greater importance to the child than the memorizing of the numerous facts it contains.

5. After the child has mastered the elementary notions of geography, it matters little to him whether the first topic presented for study is the whole earth or the city or state in which he resides, because, in reality, they differ in nothing except in size — a concept which a child can grasp or disregard as readily as an adult in map study. What is of importance to him, however, is that the topic is presented to him in bold outlines and subsequently filled in.

Method.

Elementary notions:

Where direct observation of surface conditions is impossible, clay or sand modeling should be resorted to. A child who has never seen a lake can get a pretty clear idea of it by this means, especially if the lesson is supplemented by pictures and by a comparison with some known object for the purpose of learning its size.

Map interpretation:

Have children draw a large object, as a blackboard, and call their attention to the scale; next, draw an object to an exact scale. Represent objects in the room on the plane; later, represent a river, a mountain, etc., from a clay model. Teach the cardinal points by means of the sun, and fix them on paper. Ask for exact location of any point; then teach latitude and longitude.

Causal relations:

Geography is not a compendium of isolated facts concerning our earth, but it is a study which regards its facts and phenomena as links in a long chain of causes and effects. As an example, the child, starting with location and surface as primary geographical facts, is led to perceive that they are the chief causes of climate; that location and climate influence the life of plant and animal, which in turn influence man's occupation, and these in turn affect commerce, travel, and communication.

Indeed, there is no geographical fact which will not yield to the same treatment. The general surface of a continent; its mountains, rivers, etc.; its climate; its population; the size and location of its cities; the characteristics and governments of its people; its occupations and productions; its commerce, — each of these topics can be and should be treated in its causal relation.

Ethical:

Only he who feels beauty, sympathy, and reverence, and approaches the child with gentle tact, can awaken the æsthetic feelings through the contemplation of nature or of nature's offspring — rhythmic language; can arouse sympathy through the broadening of the intellect; and can transform rational insight into reverence.

VIII. — HISTORY.

Aims:

1. *Intellectual.*
2. *Ethical.*

History is the biography of man. It not only records events which have a bearing upon his social life, but searches out their causes and effects. Such a conception of the province of history is certainly beyond the mental grasp of the young child; hence, the unfolding of the subject must be made to correspond to the development of his mental faculties.

Fundamentals.[1]

I. There are three stages in the development of the historical sense in children:

1. Narrative.
   (*a*) Biography.
   (*b*) Events.

(*a*) Sound practice demands the recognition that children's interests are most readily awakened by concrete examples. Hence, biography is chosen as the introduction to narrative which has for its basis events.

[1] Read also Fundamentals under Geography.

(*b*) The transition to events is natural, having been accomplished in accordance with the laws governing the acquisition of knowledge; while the new material for study takes into account the child's increased mental development, particularly his imagination.

2. Causal.

(*a*) Events.
(*b*) Institutions.

(*a*) Perception of causality is a more difficult process than perception of events, and requires a higher degree of mental development. The fall of Constantinople and the discovery of America are historical events which any ordinary child can grasp; but to trace out the relationship of cause and effect between them is well calculated to exercise, not merely his imagination, but his judgment and his reason as well.

(*b*) Institutions, the monuments of the world's progress, are a fitting climax in the study of history. But whether they should be dealt with as mere facts, or as the embodiments of long series of causes, is an open question, which can be determined only by the ability of the pupils to master the higher conception.

3. Ethical.

History is replete with ethical content which the teacher should avail himself of as means for the moral uplifting of the child. This is the chief aim of history teaching — not to satisfy a craving for gossip; not to

exercise the intellectual faculties; but to instill a habit of right conduct through emulation, and a love of country through appreciation of others' sacrifices and one's own duties and responsibilities.

II. The idea of concentric circles aptly characterizes the nature of the relation which exists among the materials for historical study outlined above. They are not mutually exclusive; on the contrary, they either partly or wholly include one another. Biography deals with events, with cause and effect, refers to institutions, and insinuates ethical concepts, even while it lays stress upon the individual; events are not isolated phenomena; institutions are not purposeless; nor are ethical principles self-creative.

Method.

I. Biography and Events.

We become interested in historical characters or events, either when they coincide with our sentiments, or when they are viewed in their original environments. We become still more interested in them when both these conditions are realized.

The teacher of biography and of narrative, then, must search out all the causes which have been instrumental in giving the subject of the child's study a place in history, and then arrange the facts in proper perspective. With the plan of the lesson clear in his mind, he makes his presentation, guided by the thought that the nearer biography and narrative approximate

actual occurrences, the more lifelike the characters and the events are, the greater is the appeal to the intellect and to the feelings of the listener.

But knowledge alone will not suffice the successful teacher; but back of it, interwoven with it, impelling it, must be enthusiasm — interest which has gathered a large amount of momentum. For it is the enthusiast only who can awaken in others feelings akin to his own. Such interest on the part of the teacher comes only with perfect mastery of the subject. Deep, earnest, and frequent study is essential to success in teaching history.

Minor points:

1. Mechanical aids, such as pictures, portraits, maps — everything that conduces to a reconstruction of the environment — should be employed.

2. Each biography should represent an epoch.

3. Before assigning a lesson for home study, it should be carefully explained by the teacher.

4. Before presenting a new lesson, the preceding lesson should be reviewed in order to connect the events in the mind of the child.

5. Events should be mastered by interesting repetition.

6. Important dates should be committed to memory to serve as bases for comparison.

7. The child should be trained to recite the lesson without prompting or questioning.

II. Cause and Effect.

Biography and events, which were originally taught as facts, are now made to yield to a higher process — the recognition of cause and effect. But in order that the process may be of value as an intellectual discipline, the child himself should be led to draw the inferences.

III. Ethics.

The good, the true, the beautiful, cannot be awakened into motive forces by direct teaching. Child nature resents such an attempt; while the lessons taught indirectly by biography, by events, and by institutions insinuate themselves into the mind and heart of the child and become part of his inner life — his character.

IX. — CIVICS.

Aim: Training in and for citizenship.

The teaching process, in order to be successful, must take into consideration the child to be educated, the material for his education, and the teacher, who is the medium between the two, their relative importance depending upon the aim of the lesson. Civics, which seeks primarily to impress its principles upon the moral sense of the child, must necessarily give prominence to the character of the teacher.

Fundamentals.

1. *The child.*

   (*a*) The underlying civic virtue is obedience to constituted authority; hence, train the child first to a habit of implicit obedience.

   (*b*) Obedience should aim to a rational and voluntary compliance to an inner authority; hence, rationalize the child's knowledge of law and order, broaden and deepen his sense of duty, and give impulse to his conduct.

(*c*) In intellectual education, interest is a condition to knowledge; in moral education, knowledge is a condition to interest.

(*d*) The child is in possession of all the elementary concepts which underlie this study through intercourse with those with whom he comes in contact long before he begins the study of civics.

(*e*) The principles underlying the teaching of other studies obtain here also: Faith in the concrete, reasoning, conviction, action, are the steps in the development of a civic character.

2. *Material.*

(*a*) The purposes of government.

The material must be concrete and familiar — the home, as represented by the head of the family; the school, by the teacher; the city, by the policeman and the fireman.

(*b*) A knowledge of our form of government.

The material should include the leading facts and the underlying principles of municipal, state, and federal governments.

(*c*) The duties of citizens.

A comparison with other forms of government in

order to emphasize such peculiar institutions as suffrage, primaries, and conventions.

(*d*) Historical personages and events.

3. *The teacher.*

Only the teacher who is zealous in the cause of good citizenship can arouse and maintain the child's interest in this study, can make him appreciate the blessings of our free government, can induce him to feel that he owes certain duties to his citizenship which call for cheerful responses on his part.

Method.

Model Lesson on Primaries (Rights and Duties).[1]

1. *Facts:*

(*a*) Definition.
(*b*) Date.
(*c*) Location.
(*d*) Participants.
(*e*) Purpose.

2. *Relations:*

(*a*) To the character of the candidates.
(*b*) To the character of the government.
(*c*) To the moral tone of the community.
(*d*) To the American idea of government.
(*e*) To self-respect, and to respect of others.

[1] Morally, attendance upon primaries and voting are duties legally, they are not — the more's the pity!

3. *Feelings:*

(*a*) Free government is a heritage handed down to us by centuries of oppression, suffering, and bloodshed; hence this right involves a duty, for "Eternal vigilance is the price of liberty."

(*b*) Neglect to perform the duties of citizenship results in political rings formed for selfish purposes and dominated by the one-man power — a travesty on our boasted self-government!

(*c*) Such conditions ought not to be tolerated by enlightened men, both as individuals, as members of society, and as Americans.

4. *Action:*

The teacher can only sow the seed; he must leave the rest to the future.

X. — ARITHMETIC.

Aims:

1. *Utilitarian.*
2. *Intellectual.*

General considerations:

I. As a school study, arithmetic includes:

(*a*) Examples which deal with the fundamental processes of numbers, as $2\frac{3}{4} \div 1\frac{1}{3}$.

(*b*) Problems, or questions involving numbers, in which the nature of the processes must first be determined by reasoning, as, If $1\frac{1}{3}$ yards of cloth cost $\$2\frac{3}{4}$, what will 1 yard cost?

II. The processes involved in examples may be learned in two ways:

(*a*) Through authority, as when a child is told that to divide one fraction by another, he must invert the divisor and proceed as in multiplication.

(*b*) Through induction, or the process whereby general laws are obtained through the investigation of particulars, as, for instance, when the child, in comparing the results of several divisions of fractions, formulates for himself the law of the division of fractions.

III. Arithmetical induction may be obtained in two ways:

(*a*) By investigating the results of several concrete presentations, as, for example: In dividing $\frac{2}{3}$ by $\frac{3}{5}$, the child chooses a piece of paper as a unit, measures off $\frac{2}{3}$ of it, then $\frac{3}{5}$ of it; then shows by actual measurement that the result of dividing $\frac{2}{3}$ by $\frac{3}{5}$ is $\frac{10}{9}$.

(*b*) By abstract reasoning. Thus, the answer to the above might have been obtained by reasoning in this wise: Dividing $\frac{2}{3}$ by $\frac{1}{5}$ will give $\frac{10}{3}$ as a quotient, and dividing $\frac{2}{3}$ by $\frac{3}{5}$ will give $\frac{1}{3}$ of $\frac{10}{3}$ or $\frac{10}{9}$ as a quotient.

Fundamentals.

1. The child's knowledge of the laws of arithmetic should be the result of his own inductions.

2. Induction through the concrete is the proper method for fundamental processes, and it must precede the abstract method.

3. Concrete presentation for the purpose of induction is demanded only by the processes involving whole numbers and common fractions.

4. Processes should be mastered before an attempt is made to apply them to the solution of problems.

5. Oral arithmetic is for instruction and drill; written arithmetic, for exercises that deal with large numbers.

6. Induction should be preceded by preparation and

followed by application, making the whole a process of apperception.

7. The type is of great value in arithmetic.

Method.

Fundamental operations — integers.

Guide: Concrete presentation.

Counting.

1. Counting is regarded as the initial step in arithmetic teaching, because:

(*a*) It is a process with which the child is already acquainted when he enters school.

(*b*) It is the natural basis of the fundamental operations.

2. The teaching progression: idea, word, symbol. Symbols are introduced thus early because they embody in concrete form the child's abstractions, and thereby assist in his further progress, and because of their practical utility.

Addition.

Counting (synthesis).

1. To teach the combination, three and two are five.

(*a*) Children are given, or are told to take, first, 3 sticks, and then 2 sticks. Questions follow with a view to directing the pupils' attention to the two groups and their compositions; after which they are told to

count the number of sticks in both groups (all together); and to announce the result, first as 5 sticks; later, as 3 sticks and 2 sticks are 5 sticks; and lastly, as 3 and 2 are 5.

(*b*) Represent the process and the result on the blackboard (symbol of formula).

(*c*) Drill.

2.
$$\begin{array}{r} 15 \\ +\ 8 \\ \hline 23 \end{array}$$

The child has on his desk a bundle of 10 splints, and to the right of it 5 splints, and below them a piece of paper on which he has written the number 15. After recognizing the figure which represents the bundle and the figure which represents the loose sticks, he adds 8 splints to the 5 splints, and writes the 8 below the 5. Upon counting his loose splints, he finds that he has 13 of them, 10 of which he binds into a bundle, leaving 3 loose splints. The latter he represents by a 3 placed in the units column; the bundle he transfers to the left (the transfer being marked on the paper), and the two bundles he represents by the figure 2 placed in the tens column.

Subtraction.

Counting (analysis).

$$\begin{array}{r} 35 \\ -18 \\ \hline 17 \end{array}$$

The child arranges the splints as in addition; also the paper on which the bundles are represented by 3, and the loose splints by 5. He discovers that he cannot take away 8 splints when he has only 5 splints, but that he can take a bundle and transfer it as 10 single splints to the 5 splints. He has now 2 bundles and 15 splints (a result which he marks on his paper). From the 15 splints he takes away 8 splints, leaving 7 splints (represented on the paper as 7 units); and from the 2 bundles he takes 1 bundle, leaving as a remainder 1 bundle (represented on the paper as 1 ten).

Multiplication.

Counting of equal numbers with reference to "times" (synthesis).

To teach

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline 6 \end{array}$$

Each pupil arranges 3 groups, each containing 2 objects, then counts the number of objects in the 3 groups, and announces that the 3 groups, each containing 2 objects, have in all 6 objects. Next, he writes three 2's in a column, adds them, and tells how many 2's make 6. This knowledge he now represents by the formula (briefer by comparison than that of addition)

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline 6 \end{array}$$

and commits to memory.

Division.

Counting of equal numbers with reference to "times" (analysis).

$$3\overline{)15}$$
$$5$$

Each pupil arranges his 15 objects into groups, each group containing 3 objects. He then counts the number of the groups and announces that he separated or divided the 15 objects into 5 groups, each containing 3 objects. He next writes a sufficient number of 3's in a column to add up to 15, or he subtracts successively a sufficient number of 3's from 15 to have no remainder. In either case he finds that the 3 has been involved 5 times. He now learns the formula

$$3\overline{)15}$$
$$5$$

and commits its substance to memory.

Common Fractions.

Guides:

1. *Concrete Presentation.*
2. *The Formal Steps.*

METHOD I.

Addition.

1. Preparation: Find fractional parts of whole numbers.

2. Presentation: 1. Add $\frac{1}{2}$ and $\frac{1}{3}$.

The pupil is given 6 objects, such as buttons, marbles, splints, etc., and required to find $\frac{1}{2}$ of them and $\frac{1}{3}$ of

them; then, having found that $\frac{1}{2}$ of the marbles $= 3$ marbles, and $\frac{1}{3}$ of the marbles $= 2$ marbles, with proper guidance he concludes that $\frac{1}{2}$ of the marbles $+ \frac{1}{3}$ of the marbles $= \frac{5}{6}$ of the marbles — a conclusion which he later transforms through abstraction into $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

If he proceeds likewise with the following, and comes to the conclusion that

2. $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$.
3. $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$.
4. $\frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{38}{40}$.

he has all the material necessary for the third step.

3. Comparison.

The Common Denominator.

Analysis.

Example.	Original denominators.	Common denominators.
1.	2, 3	6
2.	2, 5	10
3.	3, 4	12
4.	2, 4, 5	40[1]

[1] There is no valid reason for the custom of discontinuing thus early in favor of the least common denominator the idea that the common denominator is the product of the various denominators. The arguments in favor of the latter practice may be briefly stated as follows:

1. It does not burden the pupil with too many processes at the very outset.

2. It leaves the fractions intact.

3. The attention at the very outset is directed to the aim rather than to the preparatory matter.

4. There is an economy of energy and time — reduction of the answer is all that is needed.

4. Generalization:

The Common Denominator is the product of the several denominators.

The Numerators.

If the pupil's attention is now directed to the equations $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{5} = \frac{2}{10}$, $\frac{2}{3} = \frac{8}{12}$, etc., a vague intimation of which he already has, he will eventually discover that the ratio existing between the denominators of each equation is the same as that existing between their numerators; nor will he find great difficulty in discovering and in formulating a rule for finding the new numerators — in other words, to generalize.

5. Application (see Fundamental 6):

Processes and concepts treated incidentally in the foregoing discussion, or readily deducible from it.

1. Reduction to higher or lower terms.

Corollary (*a*).

The value of a fraction is not altered when both of its terms are multiplied or divided by the same number.

Corollary (*b*).

Factoring.

2. The need for a common denominator.

Subtraction of Fractions.

With the single exception of the actual process of subtraction, the mode of presenting this topic is the same as in addition.

Multiplication of Fractions.

Multiply $\frac{1}{2}$ by $\frac{1}{3}$.

Take 6 splints; then

$\frac{1}{2}$ of 6 splints = 3 splints.
$\frac{1}{3}$ of 3 splints = 1 splint.

Proceeding as in addition, the conclusion is reached that

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

In the same manner,

$$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}.$$
$$\frac{2}{5} \times \frac{4}{6} = \frac{8}{30}.$$

Comparison:

Original numerators.	Resulting numerators.
1, 1	1
2, 2	4
2, 4	8

Conclusion: _____

Original denominators.	Resulting denominators.
2, 3	6
3, 5	15
5, 6	30

Generalization: To multiply one fraction by another fraction, _____

Division of Fractions.

Divide $\frac{1}{3}$ by $\frac{1}{2}$.

Take 6 marbles; then

$\frac{1}{3}$ of $6 = 2$.
$\frac{1}{2}$ of $6 = 3$[1] and
$\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.

Other examples:

$\frac{3}{7} \div \frac{1}{7} = \frac{6}{7}$.
$\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$.
$\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$.

Comparison:

1. $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.
2. $\frac{3}{7} \div \frac{1}{2} = \frac{6}{7}$.
3. $\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$.
4. $\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$.

A few well-directed questions will reveal to the child that 8, the numerator of the answer to example 3, for instance, is the product of 4 and 2, and that 15, the denominator of the same answer, is the product of 3 and 5. Likewise with the other examples.

Generalization: The law governing the division of fractions.

Application:

[1] Attention must be called at this point to the change of base which takes place in multiplication, but not in division of fractions. A comparison of the two processes will make this point clear.

Multiply $\frac{1}{3} \times \frac{1}{2}$.	Divide $\frac{1}{3}$ by $\frac{1}{2}$.
$\frac{1}{3}$ of 6 [base] $= 2$.	$\frac{1}{3}$ of 6 [base] $= 2$.
$\frac{1}{2}$ of 2 [new base] $= 1$.	$\frac{1}{2}$ of 6 [same base] $= 3$.
$\therefore \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.	$\therefore \frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.

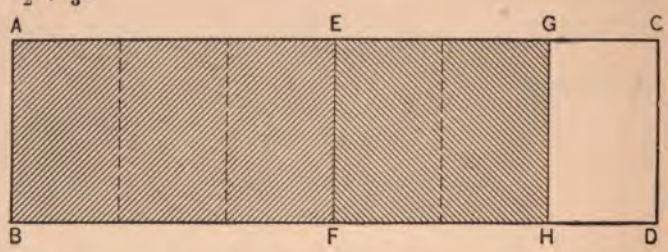
METHOD II.

(For more advanced pupils.)

The presentation step only will be discussed here. The rest have already been discussed in Method I.

Addition.

$\frac{1}{2}+\frac{1}{3}$.



AD is the unit, *AF* is $\frac{1}{2}$ of it, and *EH* is $\frac{1}{3}$ of it, and the question becomes: What part of *AD* are $AF+EH$?

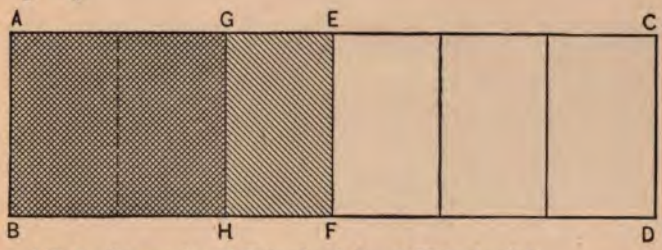
Assuming *GD*[1] as the standard of comparison, or the unit of measure, it will be found by actual measurement that $AF=3\times GD$, and $EH=2\times GD$; or that *GD* is contained 2 times in *EH*, and 3 times in *AF*. *AD* is thus seen to be divided into 6 equal parts, of which *AF* contains 3 and *EH*, 2.

So that $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

[1] In case the figure contains no exact unit of measure, one can be obtained by dividing its smallest division into 2 parts, or 3 parts, etc.

Subtraction.

$\frac{1}{2} - \frac{1}{3}$.



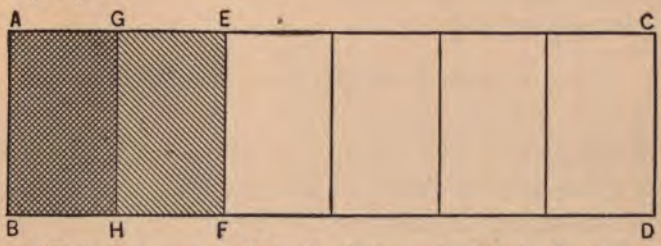
AD is the unit, *AF* is $\frac{1}{2}$ of it, *AH* is $\frac{1}{3}$ of it.

The difference between *AF* and *AH* is evidently *GF*, and the question becomes: What part of *AD* is *GF*?

Assuming *GF* as the unit of measure, and proceeding as in addition, we have $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$.

Multiplication.

$\frac{1}{2} \times \frac{1}{3}$.



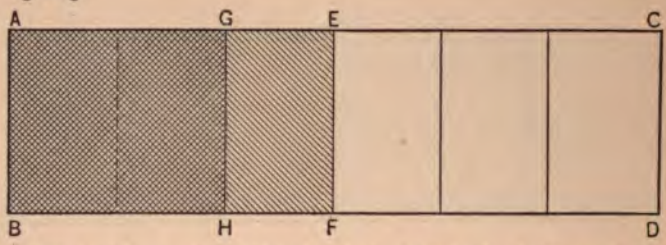
AD is the unit, *AF* is $\frac{1}{3}$ of it, and *AH* is $\frac{1}{2}$ of *AF*,[1] and the question becomes: What part of *AD* is *AH*? Evidently $\frac{1}{6}$.

So that $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

[1] See note to Method I.

Division.

$\frac{1}{2} \div \frac{1}{3}$.



AD is the unit, *AF* is $\frac{1}{2}$ of it, *AH* is $\frac{1}{3}$ of it, and the question becomes: How many times is *AH* contained in *AF*, or how does *AF* compare in size to *AH*, or what part of *AH is AF*?

Assuming *GF* as the unit of measure, and proceeding as previously, we find that *AF* is composed of 3 units, and *AH* of 2 units; so that *AF* is $\frac{3}{2}$ of *AH*, or *AH* is contained $1\frac{1}{2}$ times in *AF*.

Hence $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$.

Numeration and Notation of Decimals.

Guide: Common fractions.

Preparation: The number of places an integer of any given value requires for its notation.

Presentation: Notation of decimals, although it depends upon the device of place, can still be made the subject of inference, after the first few initial steps have been mastered by the child.

For instance, if a child has learned to recognize and to write such fractions as .3, .04, .45, he can be led to

recognize the generalizations of both processes and to apply them. Thus he has already learned:

1. The denominator of a decimal fraction is indicated by a point before the numerator.

2. That the device of place indicating the cipher is the same as in whole numbers.

3. That numbers are regarded as increasing toward the left and vice versa in a tenfold ratio.

4. That, counting the decimal point as a "place," there are just as many places needed to write a decimal fraction as a whole number of like character.

Application:

Express decimally four thousand ninety-eight hundred thousandths.

Explanation:

Since hundred-thousandths requires 6 places, and four thousand ninety-eight takes up 4 of the places, there still remain 2 places which should be filled by the decimal point and a cipher. Hence, beginning with the decimal point, the fraction is expressed as

.04098.

The method proposed here is recommended because

(*a*) It makes no distinction between whole numbers and decimal fractions in the matter of place value.

(*b*) It starts with the decimal point in notation — of great value when fractions are dictated for the purposes of addition or subtraction.

Reduction of Decimals.

Reduce $\frac{3}{8}$ to thousandths.

$\frac{3}{8}$ is the same as $3 \div 8$.

$$3 \div 8 = \frac{30}{10} \div 8 = \frac{300}{100} \div 8 = \frac{3000}{1000} \div 8 = \frac{375}{1000};$$

or, decimally, $\begin{array}{r} .375 \\ 8\overline{|3.000}. \end{array}$

Multiplication of Decimals.

Decimals being merely a device, the laws governing its fundamental operations should be taught by reference to common fractions, thus:

$$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}. \quad \therefore .1 \times .1 = .01.$$

$$\frac{24}{100} \times \frac{9}{10} = \frac{216}{1000}. \quad \therefore .24 \times .9 = .216, \text{ etc.}$$

Hence the rule: Point off in the product as many figures from the right as there are decimal figures in the multiplicand and the multiplier.

Division of Decimals.

(When the divisor is an integer.)

1. $\frac{9}{10} \div 3 = \frac{9}{10} \times \frac{1}{3} = \frac{3}{10}. \quad \therefore \begin{array}{r} .3 \\ 3\overline{|.9}. \end{array}$
2. $\frac{9}{100} \div 3 = \frac{9}{100} \times \frac{1}{3} = \frac{3}{100}. \quad \therefore \begin{array}{r} .03 \\ 3\overline{|.09}. \end{array}$
3. $\frac{9}{10000} \div 3 = \frac{9}{10000} \times \frac{1}{3} = \frac{3}{10000}. \quad \therefore \begin{array}{r} .0003 \\ 3\overline{|.0009}. \end{array}$

Conclusion: The number of decimal places in the quotient is the same as the number of decimal places in the dividend.

(When the divisor is a fraction.)

1. $9 \div \frac{3}{10} = 9 \times \frac{10}{3} = \frac{90}{3} = 90 \div 3.$

$\therefore\ .3\overline{|9} = 3\overline{|90}.$

2. $9 \div \frac{3}{100} = 9 \times \frac{100}{3} = \frac{900}{3} = 900 \div 3.$

$\therefore\ .03\overline{|9} = 3\overline{|900}.$

3. $9 \div \frac{3}{1000} = 9 \times \frac{1000}{3} = \frac{9000}{3} = 9000 \div 3.$

$\therefore\ .003\overline{|9} = 3\overline{|9000}.$

Conclusion: We can get rid of a fractional divisor by multiplying both the divisor and the dividend by the denominator of the divisor.

Deduction: Divide .009 by .03.

$.03\overline{|.009} = 3\overline{|.9}$; or, briefly, $\times 03.\overline{|\times 00.9}^{.3}.$

Proof: $\frac{9}{1000} \div \frac{3}{100} = \frac{9}{1000} \times \frac{100}{3} = \frac{3}{10}.$

Percentage.

Some of the difficulties which children encounter here will be obviated by defining "rate per cent" as *so many out of each hundred.* Thus 6% means 6 out of a hundred; 6% of \$500 means \$6 out of each \$100; 7% of 800 sheep means 7 sheep out of each group of 100 sheep.

Analysis is the basic method in percentage.

Examples:

1. Find 5% of \$800.

1% of \$800 = \$8.
5% of \$800 = \$40,

or

$$\begin{array}{r} \$8.00 \\ 5 \\ \hline \$40.00 \end{array}$$

2. What per cent of 200 is 8?

$$1\% \text{ of } 200 = 2.$$
$$\therefore 8 = 4\%.$$

Interest.

With the 360 days method, the new factor presents but little difficulty.

Example:

Interest on \$500 for 1 yr. 2 mos. 12 days at $4\frac{1}{2}\%$.

$$\frac{\$500 \times 432 \times 9}{360 \times 200} =$$

Proportion.

Problems included under this topic should be solved by analysis, as follows:

If 4 men, working 9 hrs. a day, require 2 days to build a wall 18 ft. high, how many days will 6 men, working 8 hrs. a day, require to build a wall 12 ft. high?

Analysis:

(*a*) If 4 men require 2 days,
1 man requires 4×2 days,
6 men require $\frac{1}{6}$ of 4×2 days,

or $\frac{4 \times 2}{6}$ days.

(*b*) If 6 men require $\frac{4 \times 2}{6}$ days to build 18 ft., they will require $\frac{1}{18}$ of $\frac{4 \times 2}{6}$ days to build 1 ft. and $12 \times \frac{1}{18}$ of $\frac{4 \times 2}{6}$ days to build 12 ft.

or $\frac{12 \times 4 \times 2}{18 \times 6}$ days.

(*c*) If 6 men, building a wall 12 ft. and working 9 hrs. a day, require $\frac{12 \times 4 \times 2}{18 \times 6}$ days, if they work 1 hr. a day, they will require $9 \times \frac{12 \times 4 \times 2}{18 \times 6}$ days, and working 8 hrs. a day, they will require $\frac{1}{8}$ of $9 \times \frac{12 \times 4 \times 2}{18 \times 6}$ days,

or $\frac{9 \times 12 \times 4 \times 2}{8 \times 18 \times 6}$ days.

Square Root.

First Step: Preparation.

(*a*) The notion of square root is obtained by such exercises as $3 \times 3 = 9$, $9 \times 9 = 81$, $12 \times 12 = 144$, in which the product of the two equal factors is the square, and one of the two equal factors is the square root. Square and square root are thus seen to be rela-

tive terms. Square root cannot be taught without reference to its square, nor can the latter be conceived apart from the former.

(*b*) The symbols, 5^2, $\sqrt{25}$.

(*c*) The terms "power," "involution," "evolution," "index," "radix."

(*d*) The analytic method of squaring numbers. As this topic does not engage the pupil's attention before he has reached a point in his mental development when he is already in possession of the elementary knowledge that underlies its elaboration, or before he can interpret the words of his teacher in terms of individualized or experienced notions, this method is preferable to the synthetic.

The process of squaring any number, as, for example, 25, may be presented under three different operations.

(*a*)

$$\begin{array}{r} 25 \\ \underline{25} \\ 125 \\ \underline{50} \\ 625 \end{array}$$

Which is the ordinary method of multiplication.

(*b*)

$$\begin{array}{l} 25 \\ \underline{25} \\ 25 = 5 \times 5, \text{ or } 5^2 \\ 100 = 20 \times 5 \\ 100 = 20 \times 5 \\ \underline{400 = 20 \times 20 \text{ or } 20^2} \\ 625 \end{array}$$

Which shows the operation in full.

(*c*)

$$\begin{array}{r} 20+5 \\ 20+5 \\ \hline 20\times 5+5^2 \\ 20^2+20\times 5 \\ \hline 20^2+2(20\times 5)+5^2=625 \end{array}$$

Which differs from *b* only in expressing the + symbol.

(*d*) Again,

$$\begin{array}{r} 25^2=(23+2)^2=625 \\ 23+2 \\ 23+2 \\ \hline 23\times 2+2^2 \\ 23^2+23\times 2 \\ \hline 23^2+2(23\times 2)+2^2=625 \end{array}$$

(*e*) Or

$$\begin{array}{r} 25^2=(7+18)^2=625 \\ 7+18 \\ 7+18 \\ \hline 7\times 18+18^2 \\ 7^2+7\times 18 \\ \hline 7^2+2(7\times 18)+18^2=625 \end{array}$$

That is, the square of any number = the square *of any two numbers* whose sum equals that number + twice the product of those numbers.

(*f*) Lastly,

$$\begin{array}{r} (20+5)^2=20+5 \\ 20+5 \\ \hline 20^2+20\times 5 \\ +20\times 5+5^2 \\ \hline 20^2+2(20\times 5)+5^2=625 \end{array}$$

In other words, the validity of the process and of the answer is in nowise affected when the process of multiplication begins with the numbers to the left of the plus sign.

If, for the sake of brevity, we represent the number to the left of the plus sign by x, and the number to the right of it by y, the process of involution (when the number is composed of two component parts) may be represented as,

$$(x + y)^2 = x^2 + 2\,xy + y^2.$$

A good rule to follow is not to consider the preparation completed unless the child recognizes, as the result of his own efforts:

1. That evolution is primarily a process of division;
2. That the process of extracting the square root may begin either at the right or at the left;
3. That the algebraic expression is a symbolic representation of a general truth which he can demonstrate satisfactorily; and
4. That the number may be divided into periods containing any number of figures.

Example:

Find the square root of 54756.

Beginning at the right, and pointing off periods of one figure, we have,

1.

$$2 + 5 + 20 + 41 + 166 = 234$$

$$5,\ 4,\ 7,\ 5,\ 6 = x^2 + 2\,xy + y^2$$

$$4 = y^2$$

$$52 = x^2 + 2\,xy$$

$$2 \times 2 \times 5) + 5 \times 5) \quad 45 = y^2$$

$$707 = x^2 + 2\,xy$$

$$2 \times 7 \times 20) + 20 \times 20) \quad 680 = y^2$$

$$4027 = x^2 + 2\,xy$$

$$2 \times 27 \times 41) + 41 \times 41) \quad 3895 = y^2$$

$$50132 = x^2 + 2\,xy$$

$$2 \times 68 \times 166) + 166 \times 166) \quad 50132 = x^2 + 2\,xy$$

2. Beginning at the right and pointing off periods of two figures:

$$7 + 61 + 166 = 234$$

$$5,\ 47,\ 56 = x^2 + 2\,xy + y^2$$

$$49 = y^2$$

$$4707 = x^2 + 2\,xy$$

$$2 \times 7 \times 61) + 61 \times 61) \quad 4575 = y^2$$

$$50132 = x^2 + 2\,xy$$

$$2 \times 68 \times 166) + 166 \times 166) \quad 50132 = x^2 + 2\,xy$$

Second Step: Presentation.

The following method is then shown to be the simplest.

Find the square root of 978121.

$$
\begin{array}{rl}
+9 & : \\
+80 & : \quad 989 \\
900 & : \\
\hline
978121 & \\
900 \times 900 = 810000 & \\
\hline
= 168121 & \\
2 \times 900 \times 80) + 80 \times 80) = 150400 & \\
\hline
= \ 17721 & \\
2 \times 980 \times 9) + 9 \times 9) = \ 17721 &
\end{array}
$$

Third Step: Comparison.

Comparison of several examples of like character to above will make evident the following facts:

1. That, when a quantity is expressed by an even number of figures, its square root contains one half its number of figures; but when a quantity is expressed by an odd number of figures, its square root consists of one more than one half its number of figures. This principle may also be demonstrated by involution.

2. That the subtrahend (with the possible exception of the initial and the final) contains an even number of zeros.

3. That (with the single exception of the initial subtrahend, which involves either one or two figures, de-

pending upon whether the square is made up of an even or of an odd number of figures) two figures of the minuend are successively involved in finding the square root corresponding to the given period.

4. That the sum of the products of two multiplications, both having the same number as a multiplier, equals the product of the sum of the multiplicands by the multiplier; as, for example,

$$9 \times 5 + 5 \times 5 = 14 \times 5.$$

Fourth Step: Generalization.

The aim of the lesson is best served by leading the child to frame rules, rather than to allow him to memorize the rules found in the text-book.

Fifth Step: Application.

(*a*) Find the square root of 54756 and of numerous other examples as follows:

```
        2  3   4
        5, 47, 56
        4
  43)   147
        129
 464)    1856
         1856
```

(*b*) Solve the following and similar problems:

A man owns a farm in the form of a square which contains 45 A. 25 sq. rd. How many rods in length or breadth is it?

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